Application of Derivatives

Question 1.
Find all the points of local maxima and local minima of the function $f(x) = (x - 1)^3 (x + 1)^2$
(a) 1, -1, -1/5
(b) 1, -1
(c) 1, -1/5
(d) -1, -1/5
Answer:
(a) 1, -1, -1/5

Question 2.
Find the local minimum value of the function $f(x) = \sin^4 x + \cos^4 x$, $0 < x < \frac{\pi}{2}$
(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$
(d) 0
Answer:
(b) $\frac{1}{2}$

Question 3.
Find the points of local maxima and local minima respectively for the function $f(x) = \sin 2x - x$, where
$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
(a) $\frac{-\pi}{6}, \frac{\pi}{6}$
(b) $\frac{\pi}{3}, \frac{-\pi}{3}$
(c) $\frac{-\pi}{3}, \frac{\pi}{3}$
(d) $\frac{\pi}{6}, \frac{-\pi}{6}$
Answer:
(d) $\frac{\pi}{6}, \frac{-\pi}{6}$

Question 4.
If $y = \frac{ax-b}{(x-1)(x-4)}$ has a turning point P(2, -1), then find the value of a and b respectively.
(a) 1, 2
(b) 2, 1
(c) 0, 1
(d) 1, 0
Answer:
(d) 1, 0

Question 5.
$\sin^p \theta \cos^q \theta$ attains a maximum, when θ =
(a) $\tan^{-1} \sqrt{\frac{p}{q}}$
(b) $\tan^{-1}\left(\frac{p}{q}\right)$
(c) $\tan^{-1} q$
(d) $\tan^{-1}\left(\frac{q}{p}\right)$
Answer:
(a) $\tan^{-1} \sqrt{\frac{p}{q}}$

Question 6.
Find the maximum profit that a company can make, if the profit function is given by P(x) = 41 + $24x - 18x^2$.
(a) 25
(b) 43
(c) 62
(d) 49
Answer:
(d) 49

Question 7.
If $y = x^3 + x^2 + x + 1$, then y
(a) has a local minimum
(b) has a local maximum
(c) neither has a local minimum nor local maximum
(d) None of these
Answer:
(c) neither has a local minimum nor local maximum

Question 8.
Find both the maximum and minimum values respectively of $3x^4 - 8x^3 + 12x^2 - 48x + 1$ on the

interval [1, 4].
(a) -63, 257
(b) 257, -40
(c) 257, -63
(d) 63, -257
Answer:
(c) 257, -63

Question 9.
It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Find the value of a.
(a) 100
(b) 120
(c) 140
(d) 160
Answer:
(b) 120

Question 10.
The function $f(x) = x^5 - 5x^4 + 5x^3 - 1$ has
(a) one minima and two maxima
(b) two minima and one maxima
(c) two minima and two maxima
(d) one minima and one maxima
Answer:
(d) one minima and one maxima

Question 11.
Find the height of the cylinder of maximum volume that can be is cribed in a sphere of radius a.
(a) $\frac{2a}{3}$
(b) $\frac{2a}{\sqrt{3}}$
(c) $\frac{a}{3}$
(d) $\frac{a}{3}$
Answer:
(b) $\frac{2a}{\sqrt{3}}$

Question 12.
Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm.
(a) $\frac{\pi r^3}{3\sqrt{3}}$
(b) $\frac{4\pi r^2 h}{3\sqrt{3}}$

(c) $4\pi r^3$
(d) $\frac{4\pi r^3}{3\sqrt{3}}$
Answer:
(d) $\frac{4\pi r^3}{3\sqrt{3}}$

Question 13.
The area of a right-angled triangle of the given hypotenuse is maximum when the triangle is
(a) scalene
(b) equilateral
(c) isosceles
(d) None of these
Answer:
(c) isosceles

Question 14.
Find the area of the largest isosceles triangle having perimeter 18 metres.
(a) $9\sqrt{3}$
(b) $8\sqrt{3}$
(c) $4\sqrt{3}$
(d) $7\sqrt{3}$
Answer:
(a) $9\sqrt{3}$

Question 15.
$2x^3 - 6x + 5$ is an increasing function, if
(a) $0 < x < 1$
(b) $-1 < x < 1$
(c) $x < -1$ or $x > 1$
(d) $-1 < x < -\frac{1}{2}$
Answer:
(c) $x < -1$ or $x > 1$

Question 16.
If $f(x) = \sin x - \cos x$, then interval in which function is decreasing in $0 \leq x \leq 2\pi$, is
(a) $\left[\frac{5\pi}{6}, \frac{3\pi}{4}\right]$
(b) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$
(c) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$
(d) None of these

Answer:
(d) None of these

Question 17.
The function which is neither decreasing nor increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is
(a) cosec x
(b) tan x
(c) x^2
(d) |x – 1|
Answer:
(a) cosec x

Question 18.
The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in
(a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
(b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(c) $\left(0, \frac{\pi}{2}\right)$
(d) None of these
Answer:
(d) None of these

Question 19.
The function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$ is strictly increasing for all x, if
(a) $k > \frac{3}{2}$
(b) $k < \frac{3}{2}$
(c) $k \geq \frac{3}{2}$
(d) $k \leq \frac{3}{2}$
Answer:
(a) $k > \frac{3}{2}$

Question 20.
The point on the curves $y = (x - 3)^2$ where the tangent is parallel to the chord joining (3, 0) and (4, 1) is
(a) $\left(-\frac{7}{2}, \frac{1}{4}\right)$
(b) $\left(\frac{5}{2}, \frac{1}{4}\right)$
(c) $\left(-\frac{5}{2}, \frac{1}{4}\right)$

(d) $\left(\frac{7}{2}, \frac{1}{4}\right)$
Answer:
(d) $\left(\frac{7}{2}, \frac{1}{4}\right)$

Question 21.
The slope of the tangent to the curve x = a sin t, y = a{cot t + log(tan $\frac{t}{2}$)} at the point 't' is
(a) tan t
(b) cot t
(c) tan $\frac{t}{2}$
(d) None of these
Answer:
(a) tan t

Question 22.
The equation of the normal to the curves y = sin x at (0, 0) is
(a) x = 0
(b) x + y = 0
(c) y = 0
(d) x – y = 0
Answer:
(b) x + y = 0

Question 23.
The tangent to the parabola $x^2 = 2y$ at the point (1, $\frac{1}{2}$) makes with the x-axis an angle of
(a) 0°
(b) 45°
(c) 30°
(d) 60°
Answer:
(b) 45°

Question 24.
The two curves $x^3 – 3xy^2 + 5 = 0$ and $3x^2y – y^3 – 7 = 0$
(a) cut at right angles
(b) touch each other
(c) cut at an angle $\frac{\pi}{4}$
(d) cut at an angle $\frac{\pi}{3}$
Answer:
(a) cut at right angles

Question 25.
The distance between the point (1, 1) and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$
(a) $\frac{1}{\sqrt{5}}$
(b) $\frac{-1}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{5}}$
(d) $\frac{-2}{\sqrt{5}}$
Answer:
(c) $\frac{2}{\sqrt{5}}$

Question 26.
The tangent to the curve $y = 2x^2 - x + 1$ is parallel to the line $y = 3x + 9$ at the point
(a) (2, 3)
(b) (2, -1)
(c) (2, 1)
(d) (1, 2)
Answer:
(d) (1, 2)

Question 27.
The tangent to the curve $y = x^2 + 3x$ will pass through the point (0, -9) if it is drawn at the point
(a) (0, 1)
(b) (-3, 0)
(c) (-4, 4)
(d) (1, 4)
Answer:
(b) (-3, 0)

Question 28.
Find a point on the curve $y = (x – 2)^2$. at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).
(a) (3, 1)
(b) (4, 1)
(c) (6,1)
(d) (5, 1)
Answer:
(a) (3, 1)

Question 29.
Tangents to the curve $x^2 + y^2 = 2$ at the points (1, 1) and (-1, 1) are

(a) parallel
(b) perpendicular
(c) intersecting but not at right angles
(d) none of these
Answer:
(b) perpendicular

Question 30.
If there is an error of 2% in measuring the length of a simple pendulum, then percentage error in its period is
(a) 1%
(b) 2%
(c) 3%
(d) 4%
Answer:
(a) 1%

Question 31.
If there is an error of a% in measuring the edge of a cube, then percentage error in its surface area is
(a) 2a%
(b) $\frac{a}{2}$ %
(c) 3a%
(d) None of these
Answer:
(b) $\frac{a}{2}$ %

Question 32.
If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximating error in calculating its volume.
(a) 2.46π cm^3
(b) 8.62π cm^3
(c) 9.72π cm^3
(d) 7.46π cm^3
Answer:
(c) 9.72π cm^3

Question 33.
Find the approximate value of f(3.02), where $f(x) = 3x^2 + 5x + 3$
(a) 45.46
(b) 45.76
(c) 44.76

(d) 44.46
Answer:
(a) 45.46

Question 34.
$f(x) = 3x^2 + 6x + 8, x \in R$
(a) 2
(b) 5
(c) -8
(d) does not exist
Answer:
(d) does not exist

Question 35.
The radius of a cylinder is increasing at the rate of 3 m/s and its height is decreasing at the rate of 4 m/s. The rate of change of volume when the radius is 4 m and height is 6 m, is
(a) 80π cu m/s
(b) 144π cu m/s
(c) 80 cu m/s
(d) 64 cu m/s
Answer:
(a) 80π cu m/s

Question 36.
The sides of an equilateral triangle are increasing at the rate of 2 cm/s. The rate at which the area increases, when the side is 10 cm, is
(a) $\sqrt{3}$ cm^2/s
(b) 10 cm^2/s
(c) $10\sqrt{3}$ cm^2/s
(d) $\frac{10}{\sqrt{3}}$ cm^2/s
Answer:
(c) $10\sqrt{3}$ cm^2/s

Question 37.
A particle is moving along the curve $x = at^2 + bt + c$. If $ac = b^2$, then particle would be moving with uniform
(a) rotation
(b) velocity
(c) acceleration
(d) retardation

Answer:
(c) acceleration

Question 38.
The distance 's' metres covered by a body in t seconds, is given by $s = 3t^2 - 8t + 5$. The body will stop after
(a) 1 s
(b) $\frac{3}{4}$ s
(c) $\frac{4}{3}$ s
(d) 4 s
Answer:
(c) $\frac{4}{3}$ s

Question 39.
The position of a point in time 't' is given by $x = a + bt - ct^2$, $y = at + bt^2$. Its acceleration at time 't' is
(a) b – c
(b) b + c
(c) 2b – 2c
(d) $2\sqrt{b^2 + c^2}$
Answer:
(d) $2\sqrt{b^2 + c^2}$

Question 40.
The function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing on
(a) (-1, ∞)
(b) (-∞, 0)
(c) (-∞, ∞)
(d) None of these
Answer:
(a) (-1, ∞)

Question 41.
$f(x) = \left(\frac{e^{2x}-1}{e^{2x}+1}\right)$ is
(a) an increasing function
(b) a decreasing function
(c) an even function
(d) None of these
Answer:
(a) an increasing function

Question 42.
The function f(x) = $\cot^{-1} x + x$ increases in the interval
(a) (1, ∞)
(b) (-1, ∞)
(c) (0, ∞)
(d) (-∞, ∞)
Answer:
(d) (-∞, ∞)

Question 43.
The function f(x) = $\frac{x}{\log x}$ increases on the interval
(a) (0, ∞)
(b) (0, e)
(c) (e, ∞)
(d) none of these
Answer:
(c) (e, ∞)

Question 44.
The length of the longest interval, in which the function 3 sin x – $4\sin^3 x$ is increasing, is
(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{2}$
(d) π
Answer:
(a) $\frac{\pi}{3}$

Question 45.
The coordinates of the point on the parabola $y^2 = 8x$ which is at minimum distance from the circle $x^2 + (y + 6)^2 = 1$ are
(a) (2, -4)
(b) (18, -12)
(c) (2, 4)
(d) none of these
Answer:
(a) (2, -4)

Question 46.
The distance of that point on $y = x^4 + 3x^2 + 2x$ which is nearest to the line y = 2x – 1 is
(a) $\frac{3}{\sqrt{5}}$

(b) $\frac{4}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{5}}$
(d) $\frac{1}{\sqrt{5}}$
Answer:
(d) $\frac{1}{\sqrt{5}}$

Question 47.
The function $f(x) = x + \frac{4}{x}$ has
(a) a local maxima at x = 2 and local minima at x = -2
(b) local minima at x = 2, and local maxima at x = -2
(c) absolute maxima at x = 2 and absolute minima at x = -2
(d) absolute minima at x = 2 and absolute maxima at x = -2
Answer:
(b) local minima at x = 2, and local maxima at x = -2

Question 48.
The combined resistance R of two resistors R_1 and R_2 ($R_1, R_2 > 0$) is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If $R_1 + R_2 = C$ (a constant), then maximum resistance R is obtained if
(a) $R_1 > R_2$
(b) $R_1 < R_2$
(c) $R_1 = R_2$
(d) None of these
Answer:
(c) $R_1 = R_2$

Question 49.
Find the height of a cylinder, which is open at the top, having a given surface area, greatest volume and of radius r.
(a) r
(b) 2r
(c) $\frac{r}{2}$
(d) $\frac{3\pi r}{2}$
Answer:
(a) r