Application of Derivatives

Question 1.

Find all the points of local maxima and local minima of the function $f(x) = (x - 1)^3 (x + 1)^2$ (a) 1, -1, -1/5 (b) 1, -1 (c) 1, -1/5 (d) -1, -1/5 Answer: (a) 1, -1, -1/5

Question 2.

Find the local minimum value of the function $f(x) = \sin^4 x + \cos^4 x$, $0 < x < \frac{\pi}{2}$

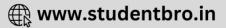
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 0 Answer: (b) $\frac{1}{2}$

Question 3.

Find the points of local maxima and local minima respectively for the function $f(x) = \sin 2x - x$, where

where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (a) $\frac{-\pi}{6}, \frac{\pi}{6}$ (b) $\frac{\pi}{3}, \frac{-\pi}{3}$ (c) $\frac{-\pi}{3}, \frac{\pi}{3}$ (d) $\frac{\pi}{6}, \frac{-\pi}{6}$ Answer: (d) $\frac{\pi}{6}, \frac{-\pi}{6}$





Question 4. If $y = \frac{ax-b}{(x-1)(x-4)}$ has a turning point P(2, -1), then find the value of a and b respectively. (a) 1, 2 (b) 2, 1 (c) 0, 1 (d) 1, 0 Answer: (d) 1, 0 Question 5. $\sin^p \theta \cos^q \theta$ attains a maximum, when $\theta =$ (a) $\tan^{-1}\sqrt{\frac{p}{q}}$ (b) $\tan^{-1}\left(\frac{p}{q}\right)$ (c) $\tan^{-1}q$ (d) $\tan^{-1}\left(\frac{q}{n}\right)$ Answer: (a) $\tan^{-1} \sqrt{\frac{p}{q}}$ Ouestion 6.

Find the maximum profit that a company can make, if the profit function is given by $P(x) = 41 + 24x - 18x^2$.

(a) 25
(b) 43
(c) 62
(d) 49
Answer:
(d) 49

Question 7.

If $y = x^3 + x^2 + x + 1$, then y (a) has a local minimum (b) has a local maximum (c) neither has a local minimum nor local maximum (d) None of these Answer: (c) neither has a local minimum nor local maximum

Question 8.

Find both the maximum and minimum values respectively of $3x^4 - 8x^3 + 12x^2 - 48x + 1$ on the





interval [1, 4]. (a) -63, 257 (b) 257, -40 (c) 257, -63 (d) 63, -257 Answer: (c) 257, -63

Question 9.

It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Find the value of a. (a) 100 (b) 120

(c) 140

(d) 160

Answer:

(b) 120

Question 10.

The function $f(x) = x^5 - 5x^4 + 5x^3 - 1$ has (a) one minima and two maxima (b) two minima and one maxima (c) two minima and two maxima (d) one minima and one maxima Answer: (d) one minima and one maxima

Question 11.

Find the height of the cylinder of maximum volume that can be is cribed in a sphere of radius a. (a) $\frac{2a}{a}$

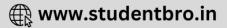
(a) $\frac{2a}{3}$ (b) $\frac{2a}{\sqrt{3}}$ (c) $\frac{a}{3}$ (d) $\frac{a}{3}$ Answer: (b) $\frac{2a}{\sqrt{3}}$

Question 12.

Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm.

(a) $\frac{\pi r^3}{3\sqrt{3}}$ (b) $\frac{4\pi r^2 h}{3\sqrt{3}}$





(c) $4\pi r^{3}$ (d) $\frac{4\pi r^{3}}{3\sqrt{3}}$ Answer: (d) $\frac{4\pi r^{3}}{3\sqrt{3}}$

Question 13.

The area of a right-angled triangle of the given hypotenuse is maximum when the triangle is (a) scalene (b) equilateral (c) isosceles (d) None of these Answer: (c) isosceles

Question 14.

Find the area of the largest isosceles triangle having perimeter 18 metres.

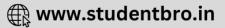
(a) $9\sqrt{3}$ (b) $8\sqrt{3}$ (c) $4\sqrt{3}$ (d) $7\sqrt{3}$ Answer: (a) $9\sqrt{3}$

Question 15. $2x^3 - 6x + 5$ is an increasing function, if (a) 0 < x < 1(b) -1 < x < 1(c) x < -1 or x > 1(d) $-1 < x < -\frac{1}{2}$ Answer: (c) x < -1 or x > 1

Question 16. If $f(x) = \sin x - \cos x$, then interval in which function is decreasing in $0 \le x \le 2\pi$, is (a) $\left[\frac{5\pi}{6}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (c) $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ (d) None of these







Answer: (d) None of these

Question 17.

The function which is neither decreasing nor increasing in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is

(a) cosec x (b) tan x (c) x^2 (d) |x - 1|Answer: (a) cosec x

Question 18.

The function $f(x) = \tan^{-1} (\sin x + \cos x)$ is an increasing function in (a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (c) $\left(0, \frac{\pi}{2}\right)$ (d) None of these Answer: (d) None of these

Question 19.

The function $f(x) = x^3 + 6x^2 + (9 + 2k)x + 1$ is strictly increasing for all x, if (a) $k > \frac{3}{2}$ (b) $k < \frac{3}{2}$ (c) $k \ge \frac{3}{2}$ (d) $k \le \frac{3}{2}$ Answer: (a) $k > \frac{3}{2}$

Question 20.

The point on the curves $y = (x - 3)^2$ where the tangent is parallel to the chord joining (3, 0) and (4, 1) is

(a) $\left(-\frac{7}{2},\frac{1}{4}\right)$ (b) $\left(\frac{5}{2},\frac{1}{4}\right)$ (c) $\left(-\frac{5}{2},\frac{1}{4}\right)$



(d) $\left(\frac{7}{2}, \frac{1}{4}\right)$ Answer: (d) $\left(\frac{7}{2}, \frac{1}{4}\right)$

Question 21.

The slope of the tangent to the curve $x = a \sin t$, $y = a \{\cot t + \log(\tan \frac{t}{2})\}$ at the point 't' is (a) tan t (b) cot t (c) $\tan \frac{t}{2}$ (d) None of these Answer: (a) tan t Question 22.

The equation of the normal to the curves $y = \sin x$ at (0, 0) is (a) x = 0(b) x + y = 0(c) y = 0(d) x - y = 0Answer: (b) x + y = 0

Question 23.

The tangent to the parabola $x^2 = 2y$ at the point $(1, \frac{1}{2})$ makes with the x-axis an angle of (a) 0° (b) 45° (c) 30° (d) 60° Answer: (b) 45° Question 24. The two curves $x^3 - 3xy^2 + 5 = 0$ and $3x^2y - y^3 - 7 = 0$ (a) cut at right angles (b) touch each other (c) cut at an angle $\frac{\pi}{4}$

(d) cut at an angle $\frac{\pi}{3}$ Answer: (a) cut at right angles



Question 25.

The distance between the point (1, 1) and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point x = 0

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{-1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{-2}{\sqrt{5}}$ Answer: (c) $\frac{2}{\sqrt{5}}$

Question 26.

The tangent to the curve $y = 2x^2 - x + 1$ is parallel to the line y = 3x + 9 at the point (a) (2, 3) (b) (2, -1) (c) (2, 1) (d) (1, 2) Answer: (d) (1, 2)

Question 27.

The tangent to the curve $y = x^2 + 3x$ will pass through the point (0, -9) if it is drawn at the point (a) (0, 1)(b) (-3, 0)(c) (-4, 4)(d) (1, 4)Answer: (b) (-3, 0)

Question 28.

Find a point on the curve $y = (x - 2)^2$. at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4). (a) (3, 1) (b) (4, 1) (c) (6,1) (d) (5, 1) Answer: (a) (3, 1)

Question 29. Tangents to the curve $x^2 + y^2 = 2$ at the points (1, 1) and (-1, 1) are







(a) parallel

(b) perpendicular

(c) intersecting but not at right angles

(d) none of these

Answer:

(b) perpendicular

Question 30.

If there is an error of 2% in measuring the length of a simple pendulum, then percentage error in its period is

(a) 1%

(b) 2%

(c) 3%

(d) 4%

Answer:

(a) 1%

Question 31.

If there is an error of a% in measuring the edge of a cube, then percentage error in its surface area is

(a) 2a%(b) $\frac{a}{2}$ % (c) 3a%(d) None of these Answer:

(b) $\frac{a}{2}$ %

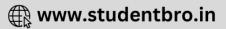
Question 32.

If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximating error in calculating its volume.

(a) $2.46\pi \text{ cm}^3$ (b) $8.62\pi \text{ cm}^3$ (c) $9.72\pi \text{ cm}^3$ (d) $7.46\pi \text{ cm}^3$ Answer: (c) $9.72\pi \text{ cm}^3$

Question 33. Find the approximate value of f(3.02), where $f(x) = 3x^2 + 5x + 3$ (a) 45.46 (b) 45.76 (c) 44.76





(d) 44.46 Answer: (a) 45.46 Question 34. $f(x) = 3x^2 + 6x + 8, x \in R$ (a) 2 (b) 5 (c) -8 (d) does not exist Answer: (d) does not exist

Question 35.

The radius of a cylinder is increasing at the rate of 3 m/s and its height is decreasing at the rate of 4 m/s. The rate of change of volume when the radius is 4 m and height is 6 m, is (a) 80π cu m/s (b) 144π cu m/s (c) 80 cu m/s (d) 64 cu m/s Answer: (a) 80π cu m/s

Question 36.

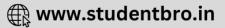
The sides of an equilateral triangle are increasing at the rate of 2 cm/s. The rate at which the area increases, when the side is 10 cm, is

(a) $\sqrt{3} \text{ cm}^2/\text{s}$ (b) 10 cm²/s (c) 10 $\sqrt{3}$ cm²/s (d) $\frac{10}{\sqrt{3}}$ cm²/s Answer: (c) 10 $\sqrt{3}$ cm²/s

Question 37. A particle is moving along the curve $x = at^2 + bt + c$. If $ac = b^2$, then particle would be moving with uniform (a) rotation

- (b) velocity
- (c) acceleration
- (d) retardation





Answer: (c) acceleration

Question 38.

The distance 's' metres covered by a body in t seconds, is given by $s = 3t^2 - 8t + 5$. The body will stop after

(a) 1 s (b) $\frac{3}{4}$ s (c) $\frac{4}{3}$ s (d) 4 s Answer: (c) $\frac{4}{3}$ s

Question 39.

The position of a point in time 't' is given by $x = a + bt - ct^2$, $y = at + bt^2$. Its acceleration at time 't' is

(a) b - c(b) b + c(c) 2b - 2c(d) $2\sqrt{b^2 + c^2}$ Answer: (d) $2\sqrt{b^2 + c^2}$

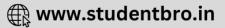
Question 40. The function $f(x) = \log (1 + x) - \frac{2x}{2+x}$ is increasing on (a) (-1, ∞) (b) (- ∞ , 0) (c) (- ∞ , ∞) (d) None of these Answer: (a) (-1, ∞)

Question 41. $f(x) = \left(rac{e^{2x}-1}{e^{2x}+1}
ight)$ is

(a) an increasing function
(b) a decreasing function
(c) an even function
(d) None of these
Answer:
(a) an increasing function



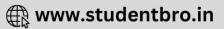




Question 42. The function $f(x) = \cot^{-1} x + x$ increases in the interval (a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(0, \infty)$ $(d)(-\infty,\infty)$ Answer: (d) $(-\infty, \infty)$ Question 43. The function $f(x) = \frac{x}{\log x}$ increases on the interval (a) $(0, \infty)$ (b) (0, e)(c) (e, ∞) (d) none of these Answer: (c) (e, ∞) Question 44. The length of the longest interval, in which the function $3 \sin x - 4 \sin^3 x$ is increasing, is (a) $\frac{\pi}{3}$ (b) $\frac{\frac{\pi}{2}}{\frac{2}{3\pi}}$ (c) $\frac{3\pi}{2}$ (d) π Answer: (a) $\frac{\pi}{3}$ Question 45. The coordinates of the point on the parabola $y^2 = 8x$ which is at minimum distance from the circle $x^{2} + (y + 6)^{2} = 1$ are (a)(2, -4)(b) (18, -12) (c)(2,4)(d) none of these Answer: (a)(2, -4)Question 46. The distance of that point on $y = x^4 + 3x^2 + 2x$ which is nearest to the line y = 2x - 1 is

(a) $\frac{3}{\sqrt{5}}$





(b) $\frac{4}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{5}}$ Answer: (d) $\frac{1}{\sqrt{5}}$

Question 47. The function $f(x) = x + \frac{4}{x}$ has (a) a local maxima at x = 2 and local minima at x = -2(b) local minima at x = 2, and local maxima at x = -2(c) absolute maxima at x = 2 and absolute minima at x = -2(d) absolute minima at x = 2 and absolute maxima at x = -2Answer: (b) local minima at x = 2, and local maxima at x = -2

Question 48.

The combined resistance R of two resistors R_1 and $R_2 (R_1, R_2 > 0)$ is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If $R_1 + R_2 = C$ (a constant), then maximum resistance R is obtained if (a) $R_1 > R_2$ (b) $R_1 < R_2$ (c) $R_1 = R_2$ (d) None of these Answer: (c) $R_1 = R_2$

Question 49.

Find the height of a cylinder, which is open at the top, having a given surface area, greatest volume and of radius r.

(a) r (b) 2r (c) $\frac{r}{2}$ (d) $\frac{3\pi r}{2}$ Answer: (a) r

